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Chiral symmetry and hyperfine $q\bar{q}$ splittings

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Abstract. We review theoretical calculations for the pseudoscalar-vector meson hyperfine splitting with no open flavor and also report a many body field theoretical effort to assess the impact of chiral symmetry in the choice of effective potentials for relativistic quark models. Our calculations predict the missing η_b meson to have mass near 9400 MeV.

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Shortly after the discovery of the J/ψ it was understood that a rich spectroscopy of new mesons awaited classification. In this task the constituent quark model was a useful tool by providing a simple periodic table where spectra and various radiative decays could be correlated with the help of a modest number of parameters. In this picture vector mesons are a $q\bar{q}$ pair, in an S or D wave, with spins parallel giving total angular momentum $J = 1$. Pseudoscalar mesons correspond to the $J = 0$ ground state with S-wave $q\bar{q}$ pairs spins antialigned. Ignoring the D-wave component, the only difference between both systems is the relative alignment of the spins. Any spectroscopic mass splitting can conveniently be incorporated in the quark model with a term, $A\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}$. that is reminiscent of the electron-nucleus spin-spin coupling, whence the name "hyperfine". This was immediately noted [1] by Appelquist *et al.* and they predicted a charmonium splitting, $\Delta M(J/\psi - \eta_c)$, of about 65 MeV. They extracted the amplitude A by estimating the J/ψ electron-positron width, $\Gamma_{e^-e^+}$, to be 4 keV. Using the currently accepted value of 5.3 keV, the splitting would be about 84 MeV, or about a factor of 2 smaller than the accepted experimental value of $3120 - 2980 = 140$ MeV. The need for a confining potential [2] was soon understood, and calculations (by Appelquist and Politzer, and independently Schnitzer [3]) including a confining strength yielded a larger splitting ($40 - 80$ MeV) than purely coulombic potentials ($15 - 20$ MeV).

In retrospective we see that many of the early models utilized scalar confining potentials, which provided a good spin-orbit coupling and radial excitations, but underestimated the hyperfine splittings. In the early eighties, and with the η_c experimental state now known, this splitting became a benchmark for new model calculations [4, 5, 6, 7] which now predicted the corresponding splitting in bottomonium. The variation in these predictions is summarized in Table 1. Subsequently, further progress was achieved through improved, renormalized non-relativistic perturbative QCD calculations (NRPQCD) [8, 9] which described bottomonium as a non-relativistic system. However, the calculated radii of most $b\bar{b}$ states are too large indicating that a coulombic description, where the relativistic splittings scale linearly with the quark mass, is not reliable and that strong interactions still induce important corrections at this scale [10]. Nevertheless, approximate ground state descriptions are feasible and useful for extracting c and b quark masses.

Non-perturbative lattice calculations with large error bars have also been performed [11, 12] for bottomonium which yield about half, or less, the hyperfine splitting exhibited in charmonoium This again indicates the system is not fully coulombic since the splitting is not proportional to the quark mass.

Extending the analysis accurately to the π-ρ system is not currently feasible for either the perturbative or lattice approach. Thus one still relies on constituent models where the hyperfine splitting has a $1/M^2$ dependence on the constituent quark mass [4]. This can describe the large π-ρ splitting but not simultaneously the hadron scattering phase shifts [13]. On the other hand, we know that the pion's mass is very low because of its Goldstone boson

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Table 1. Various existing predictions for the splitting between vector and pseudoscalar $b\bar{b}$ mesons. CQM stands for Constituent Quark Model. Units are MeV

Date, Authors	Model	Splitting
1983 Godfrey & Isgur	CQM	60
1983 McClary & Byars	CQM	101
1985 Igi & Ono	CQM coulombic	60
1985 Igi & Ono	CQM log running	90
1989 Song	CQM	55
1994 Eichten & Quigg	CQM Cornell	141
1994 Eichten & Quigg	CQM various	87/65/64
1994(98) Davies *et al.*	Lattice	30-50
1998 Pineda & Yndurain	NRQCD	47(20)
2000 Lengyel *et al.*	CQM	46
2003 Ebert *et al.*	CQM	60

nature from spontaneous chiral symmetry breaking. Thus it is natural to seek a field-theoretical formulation of the quark model which implements chiral symmetry consistently. Such an approach would predominantly attribute the hyperfine splitting in light mesons to chiral symmetry. This permits using a more moderate hyperfine potential to then decribe the smaller splittings which are exhibited in light meson excited states and heavy mesons, both of which are not governed by chiral symmetry. Thus we consider the Hamiltonian (inspired in Coulomb-gauge QCD)

$$H_{eff} = T + V_C + V_T \tag{1}$$

$$T = \int d\mathbf{x} \Psi^\dagger(\mathbf{x})(-i\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + m_q\beta)\Psi(\mathbf{x}) \tag{2}$$

$$V_C = -\frac{1}{2}\int d\mathbf{x}d\mathbf{y}\rho^a(\mathbf{x})\hat{V}(|\mathbf{x}-\mathbf{y}|)\rho^a(\mathbf{y}) \tag{3}$$

$$V_T = \frac{1}{2}\int d\mathbf{x}d\mathbf{y} J_i^a(\mathbf{x})J_j^a(\mathbf{y}) \times \left(\delta_{ij} - \frac{\nabla_i\nabla_j}{\nabla^2}\right)_{\mathbf{x}} \hat{U}(|\mathbf{x}-\mathbf{y}|) \ . \tag{4}$$

Here $\rho^a = \Psi^\dagger T^a \Psi$ is the quark color density. This Hamiltonian has been diagonalized previously [14] for $V_T = 0$ in the Bardeen-Cooper-Schrieffer (BCS) approximation for the vacuum. These earlier studies of the gap equation determined that the dynamical chiral symmetry breaking from only a longitudinal potential is relatively small and yields a low condensate $\langle\bar{\Psi}\Psi\rangle_0 \simeq -(100\ MeV)^3$.

On the opposite limit, calculations for high quark masses using the Tamm-Dancoff (TDA) and Random Phase (RPA) approximations for both harmonic oscillator [15,16] and linear potentials [17] produce almost degenerate pseudoscalar and vector meson ground states.

They are thus unable to describe the charmonium hyperfine splitting although they can reproduce the π-ρ splitting by sufficiently lowering the quark mass according to Thouless theorem in the RPA. More recently a study [18] implementing chiral symmetry used $V_C = 0$ and a contact potential for V_T to obtain a link with transverse one-gluon exchange, which is suppressed in our approach by the large gluon mass gap [19]. Because that model does not include radial excitations or confinement, we have generalized [20] the treatment by employing both a coulomb instantaneous interaction and a transverse hyperfine potential. For the longitudinal coulomb interaction we utilize a potential derived [21] from QCD through a BCS truncation of the gluon sector, represented in momentum space by

$$\hat{V}(p) = C(p) \equiv -\frac{8.07}{p^2}\frac{\log^{-0.62}\left(\frac{p^2}{m_g^2}+0.82\right)}{\log^{0.8}\left(\frac{p^2}{m_g^2}+1.41\right)} \quad \text{for } p > m_g$$

$$\hat{V}(p) = -\frac{12.25\, m_g^{1.93}}{p^{3.93}} \quad \text{for } p < m_g \ . \tag{5}$$

This is numerically close to the standard Coulomb + linear potential. The transverse potential, due to non-explicit Lorentz covariance in Coulomb gauge QCD, can be different. Since this term has not been studied theoretically, we proceed phenomenologically and choose the same Coulomb tail as in (5). It is then matched at low momentum to a Yukawa representing a massive gluon exchange which emerges from intermediate hybrid states in the Fock space truncation.

Thus we take

$$\hat{U}(p) = C(p) \quad \text{for } p > m_g \tag{6}$$

$$\hat{U}(p) = -\frac{C_h}{p^2+m_g^2} \quad \text{for } p < m_g \ .$$

The constant C_h matches the potential continuously at the m_g scale. Thus the only free potential parameter is m_g which determines simultaneously the strength of the confining term and the logarithmic one-loop running of both $\hat{U}$ and $\hat{V}$. We adopt $m_g = 600\ MeV$ and investigate alternative transverse potentials in a more detailed publication [20].

Calculating the resulting gap equation at zero quark mass we find a sizeable increase of the BCS quark condensate, to $-(178\ MeV)^3$, which is now closer to the phenomenologically accepted values (this quantity is sensitive to the high energy behaviour of the potential) as previously noted by Lagae [22]. For this chiral limit the calculated pion mass is effectively zero (numerically a fraction of an MeV) and the ρ mass is about 780 MeV. For the vector mesons we include coupled S and D wave channels, since the Hamiltonian of (1) contains a tensor interaction.

Upon increasing the quark mass, the pion mass increases rapidly in the RPA whereas the ρ mass only growes slowly yielding the hyperfine splitting plotted in Fig. 1 for various meson masses. This figure presents our preliminary results and reflects the success of this approach which incorporates chiral symmetry and is simultaneously applicable to a wide range of quark masses.

For the same model parameters, we also predict the mass of the missing η_b state, a most important issue in hadronic spectroscopy [23]. We concur with NRPQCD and lattice studies but predict a slightly larger splitting, (see below) of about 60 MeV. Subtracting this from the

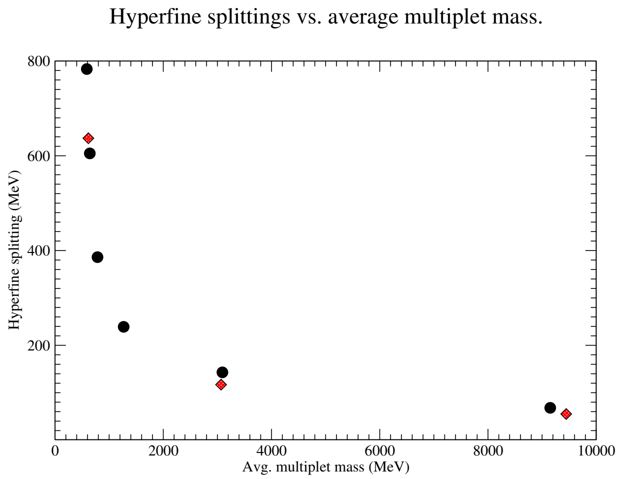

Fig. 1. RPA ground state hyperfine splittings, $M(1^{--}) - M(0^{-+})$, versus the average multiplet mass $(3M(1^{--}) + M(0^{-+}))/4$ (circles). The three diamonds represent the observed π-ρ and η_c-J/ψ and the NRPQCD η_b-Υ splittings

$\Upsilon(9460)$ mass yields $\eta_b(9400)$. This decreasing hyperfine strength trend with increasing quark mass (see Fig. 1) indicates that the potential is not yet scaleless. Note that in both PQCD and our approach (see 5) a hadron scale appears logarithmically in the coupling constant. Also for bottomonium there is a small difference between the RPA and TDA hyperfine splittings since the TDA η_b mass is about 30 MeV lower than in the RPA. While insignificant when compared to the $\Upsilon(9460\ MeV)$ mass, it should be accurately included when evaluating a small hyperfine splitting. Non-chiral preserving models, such as those based on Schrödinger's equation, will thus underestimate the splitting by at least this 30 MeV. Although this is currently comparable to the quoted errors in both NRPQCD and lattice calculations $(20-30\ MeV)$, it may become an issue in the future.

Finally, it is noteworthy that our approach naturally extends to radial excitations. For the $\psi(2S)-\eta_c(2S)$ splitting we obtain 56 MeV, in agreement with the BELLE [24] result.

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References

1. T. Appelquist et al.: Phys. Rev. Lett. **34**, 365 (1975)
2. E. Eichten et al.: Phys. Rev. Lett. **34**, 369 (1975)
3. H.J. Schnitzer: Phys. Rev. D **13**, 74 (1975)
4. S. Godfrey and N. Isgur: Phys. Rev. D **32**, 184 (1985)
5. R. McClary and N. Byars: Phys. Rev. D **28**, 1692 (1983)
6. E.J. Eichten and C. Quigg: hep-ph/9402210
7. K. Igi and S. Ono: Phys. Rev. D **33**, 3349 (1986)
8. A. Pineda and F.J. Yndurain: Phys. Rev. D **61**, 077505 (2000)
9. N. Brambilla, Y. Sumino, and A. Vairo: Phys. Lett. B **513**, 381 (2001)
10. H. Leutwyler: Phys. Lett. B **98**, 447 (1981)
11. C.T.H. Davies et al.: hep-lat/9406017
12. C.T.H. Davies et al.: hep-lat/9802024
13. P. Bicudo and J.E. Ribeiro: Z. Phys. C **38**, 453 (1988)
14. S.L. Adler and A.C. Davis: Nucl. Phys. B **244**, 460 (1984) A. Le Yaouanc et al.: Phys. Rev. D **29**, 1233 (1984); R. Alkofer and P.A. Amundsen: Nucl. Phys. B **306**, 305 (1988); P. Bicudo and J.E. Ribeiro: Phys. Rev. D **42**, 5 (1990); A.P. Szczepaniak and E.S. Swanson: Phys. Rev. D **55**, 3987 (1997)
15. A. Le Yaouanc et al.: Phys. Rev. D **31**, 137 (1985)
16. P. Bicudo and J.E. Ribeiro: Phys. Rev. D **42**, 1625 (1990)
17. F.J. Llanes-Estrada and S.R. Cotanch: Phys. Rev. Lett. **84**, 1102 (2000); Nucl. Phys. A **697**, 303 (2002)
18. A.P. Szczepaniak and E.S. Swanson: Phys. Rev. Lett. **87**, 072001 (2001)
19. A.P. Szczepaniak, E.S. Swanson, C.R. Ji, and S.R. Cotanch: Phys. Rev. Lett. **7**6, 2011 (1996)
20. F.J. Llanes-Estrada, S.R. Cotanch, A.P. Szczepaniak, and E.S. Swanson: manuscript in preparation
21. A.P. Szczepaniak and E.S. Swanson: Phys. Rev. D **62**, 094027 (2000)
22. J.F. Lagae: Phys. Rev. D **45**, 317 (1992)
23. S. Godfrey and J.L. Rosner: hep-ph/0210399; S. Godfrey: hep-ph/0210400; M. Shapkin has reported at this conference on continuing searches with negative results so far
24. S.-K. Choi et al. (BELLE collaboration): Phys. Rev. Lett. **89**, 102001 (2002); erratum-ibid. **89** 129901 (2002)
25. V. Lengyel et al.: hep-ph/0007084
26. D. Ebert, R.N. Faustov, V.O. Galkin: Phys. Rev. D **62**, 034014 (2000); hep-ph/0304227; hep-ph/0308150